

Implications of Bi-Large Neutrino Mixing on GUTs

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Under the assumptions that 1) the quark/lepton mass matrices take Froggatt-Nielsen's factorized power form $\lambda^{\psi_i+\psi_j}$ with anomalous $U(1)$ charges ψ_i , and 2) the $U(1)$ charges ψ_i respect the $SU(5)$ GUT structure, we show that the quark mass data and the mass-squared difference ratio of solar and atmospheric neutrinos, as inputs, necessarily imply that both the 1-2 and 2-3 mixings in the MNS matrix U_{MNS} are large. This analysis also gives a prediction that $U_{e3} \equiv (U_{\text{MNS}})_{13}$ is of order $\lambda \sim (0.1 - 0.5)$. We also add an argument that E_6 GUT is favored.

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Existence of a certain grand unified theory (GUT) beyond the standard model is guaranteed by i) the anomaly cancellation between quarks and leptons and ii) the unification of the gauge coupling constants at energy scale around $\mu \sim 10^{16}$ GeV. The strongest candidate for the unified gauge group is E_6 , which is not only suggested by string theory but also unique in the property that it is the maximal safe simple group allowing complex representations in the E -series; $E_3 = SU(3) \times SU(2)$, $E_4 = SU(5)$, $E_5 = SO(10)$, E_6 , E_7 , E_8 . [1]

On the other hand, the present neutrino data [2–5] show the following particular facts:

1. Bi-large mixing;

$$\sin^2 2\theta_{12} \sim 0.8, \quad \sin^2 2\theta_{23} \sim 1. \quad (1)$$

2. Mass-squared difference ratio of solar (\odot) to atmospheric (\oplus) neutrinos;

$$\frac{\Delta m_{\odot}^2}{\Delta m_{\oplus}^2} \sim \frac{7 \times 10^{-5} \text{ eV}^2}{2 \times 10^{-3} \text{ eV}^2} \sim \lambda^2, \quad (2)$$

where λ defined below is a quantity of magnitude $\lambda \sim 0.22$.

These show a sharp contrast to the quark sector, in which the mixings are very small and the mass spectrum is hierarchical. The mutual relations of the masses and mixing angles between quarks and leptons/neutrinos will be great clues for the GUTs.

The purpose of this paper is to analyze the implications of these neutrino data on the possible GUTs. [6] We analyze these data first assuming, as an working hypothesis, a supersymmetric $SU(5)$ GUT and the Froggatt-Nielsen mechanism [7] for generating hierarchical quark/lepton masses. The latter mechanism utilizes a (usually anomalous) $U(1)_X$ charge to generate effective Yukawa couplings via higher dimensional interaction terms in the superpotential of the form

$$y \Psi_i \Psi_j H \left(\frac{\Theta}{M_{\text{Pl}}} \right)^{\psi_i+\psi_j+h}, \quad (3)$$

where the ‘pre-Yukawa’ coupling constants y can in principle depend on the generation label i, j but are here assumed to be all of order 1 and so are denoted by y collectively. Θ is the Froggatt-Nielsen field carrying the $U(1)_X$ charge -1 and the $U(1)_X$ charges of the Higgs chiral superfield H and matter chiral superfields Ψ_i ($i = 1, 2, 3$) are denoted by the corresponding lower-case letters:

$$X(\Theta) = -1, \quad X(H) = h, \quad X(\Psi_i) = \psi_i (\geq 0). \quad (4)$$

After the Froggatt-Nielsen field Θ develops a vacuum expectation value (VEV) $\langle \Theta \rangle$, which is assumed to be smaller than the Planck scale by a factor of Cabibbo angle θ_C

$$\frac{\langle \Theta \rangle}{M_{\text{Pl}}} \equiv \lambda \sim 0.22 \simeq \sin \theta_C, \quad (5)$$

the effective Yukawa couplings induced from Eq. (3) are given by

$$y_{ij}^{\text{eff}} = y \times \lambda^{\psi_i+\psi_j+h} = \mathcal{O}(1) \times \lambda^{\psi_i+\psi_j+h}. \quad (6)$$

That is, suppressing the $\mathcal{O}(1)$ coefficients henceforth, and distinguishing the right-handed and left-handed matter superfields Ψ_i^{R} and Ψ_i^{L} , the mass matrix M takes the form

$$M = yv\lambda^h \times_{i>} \begin{pmatrix} & \overset{j}{\downarrow} \\ \lambda^{\psi_i^{\text{R}}+\psi_j^{\text{L}}} \end{pmatrix} \quad (7)$$

with $\langle H \rangle = v$ and ψ_i^{R} and ψ_j^{L} denoting the $U(1)_X$ charges of Ψ_i^{R} and Ψ_j^{L} , respectively. Thus, in this Froggatt-Nielsen mechanism, the hierarchical mass structure can be explained by the difference of the $U(1)_X$ charges $\psi_i^{\text{R,L}}$ of the matter fields. Note that this type of ‘factorized’ mass matrix can be diagonalized as

$$VMU^\dagger = M^{\text{diag}}. \quad (8)$$

by the unitary matrices U and V taking also similar power forms:

$$U \sim \left(\lambda^{|\psi_i^L - \psi_j^L|} \right), \quad V \sim \left(\lambda^{|\psi_i^R - \psi_j^R|} \right). \quad (9)$$

We assume $SU(5)$ structure for the quark and lepton matter contents and the $U(1)_X$ charge assignment for them. Then, the higher dimensional Yukawa couplings responsible for the up-quark sector masses, which are invariant under $SU(5)$ and $U(1)_X$, are given by:

$$y_u \Psi_i(\mathbf{10}) \Psi_j(\mathbf{10}) H_u(\mathbf{5}) \left(\frac{\Theta}{M_{\text{Pl}}} \right)^{\psi_i(\mathbf{10}) + \psi_j(\mathbf{10}) + h_u}. \quad (10)$$

After the VEV (5) is developed, these yield the effective Yukawa coupling constants

$$y_{u_{ij}}^{\text{eff}} = y_u \times \lambda^{\psi_i(\mathbf{10}) + \psi_j(\mathbf{10}) + h_u}. \quad (11)$$

In order for these to reproduce the observed up-type quark mass hierarchy structure

$$m_t : m_c : m_u = 1 : \lambda^4 : \lambda^7, \quad (12)$$

we are led to choose the following values for the $U(1)_X$ charges of three generation $\Psi_i(\mathbf{10})$ fermions taking $h_u = 0$ for simplicity: [8]

$$(\psi_1(\mathbf{10}), \psi_2(\mathbf{10}), \psi_3(\mathbf{10})) = (3, 2, 0) \quad (13)$$

Next we consider the mass matrices of down-type quarks and charged leptons which come from the couplings

$$\begin{aligned} y_d \Psi_i(\mathbf{10}) \Psi_j(\mathbf{5}^*) H_d(\mathbf{5}^*) \left(\frac{\Theta}{M_{\text{Pl}}} \right)^{\psi_i(\mathbf{10}) + \psi_j(\mathbf{5}^*) + h_d} \\ \rightarrow y_{d_{ij}}^{\text{eff}} = y_d \times \lambda^{\psi_i(\mathbf{10}) + \psi_j(\mathbf{5}^*) + h_d}. \end{aligned} \quad (14)$$

Note that this yields the transposed relation between the down-type quark mass matrix M_d and the charged lepton one M_l : $M_d^T \sim M_l$. This is because the $\Psi_i(\mathbf{5}^*)$ multiplets contain the right-handed component d^c for the down-type quarks while the left-handed component l for the charged leptons. Therefore the unitary matrices for diagonalizing those mass matrices, satisfy the relations

$$\begin{cases} V_d M_d U_d^\dagger = M_d^{\text{diag.}} \\ V_l M_l U_l^\dagger = M_l^{\text{diag.}} \end{cases} \rightarrow \begin{cases} V_l = U_d^* \\ V_d = U_l^* \end{cases}, \quad (15)$$

so that we have $U_d^*(M_l \sim M_d^T)U_l^\dagger = \text{diag.}$ with

$$U_d \sim \left(\lambda^{|\psi_i(\mathbf{10}) - \psi_j(\mathbf{10})|} \right), \quad U_l \sim \left(\lambda^{|\psi_i(\mathbf{5}^*) - \psi_j(\mathbf{5}^*)|} \right). \quad (16)$$

That is, the mass matrix takes the form

$$M_d^T \sim M_l \sim y_d v \lambda^{h_d} \times \left(\lambda^{\psi_i(\mathbf{10}) + \psi_j(\mathbf{5}^*)} \right). \quad (17)$$

In order for this M_d to reproduce the mass ratio of the top and bottom quarks

$$\frac{m_b}{m_t} \sim_{\text{exp.}} \lambda^{2-3} \quad (18)$$

we take $\psi_3(\mathbf{5}^*) = 2 - h_d$. Further, to reproduce the down-type quark mass hierarchy

$$m_b : m_s : m_d =_{\text{exp.}} 1 : \lambda^2 : \lambda^4, \quad (19)$$

we take $\psi_2(\mathbf{5}^*) = \psi_1(\mathbf{5}^*) - 1 = \psi_3(\mathbf{5}^*)$, so that

$$(\psi_1(\mathbf{5}^*), \psi_2(\mathbf{5}^*), \psi_3(\mathbf{5}^*)) = (3 - h_d, 2 - h_d, 2 - h_d), \quad (20)$$

and the mass matrix (17) now reduces to

$$M_d^T \sim M_l \sim y_d v \lambda^2 \times \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix} \quad (21)$$

This form of mass matrix is called lopsided.

Mixing matrices in the quark sector and lepton sector are called Cabibbo-Kobayashi-Maskawa (CKM) and Maki-Nakagawa-Sakata (MNS) [9] matrices, respectively, and they are defined by

$$U_{\text{CKM}} = U_u U_d^\dagger, \quad U_{\text{MNS}} = U_l U_\nu^\dagger. \quad (22)$$

In our case both U_u and U_d takes the form $U_u \sim U_d \sim (\lambda^{|\psi_i(\mathbf{10}) - \psi_j(\mathbf{10})|})$, so that the CKM matrix, generally, also has the same form

$$U_{\text{CKM}} \sim (\lambda^{|\psi_i(\mathbf{10}) - \psi_j(\mathbf{10})|}) \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad (23)$$

agreeing perfectly the experimental data. For the charged lepton sector we have

$$U_l \sim (\lambda^{|\psi_i(\mathbf{5}^*) - \psi_j(\mathbf{5}^*)|}) \sim \begin{pmatrix} 1 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}. \quad (24)$$

If the mixing matrix U_ν in neutrino sector is ~ 1 , this beautifully explains the observed large 2-3 neutrino mixing! However, this alone fails in explaining the large 1-2 mixing. We thus have to discuss the neutrino mixing matrix U_ν now.

Generally in GUTs, there appear some right-handed neutrinos $\Psi_I(\mathbf{1}) = \nu_{RI}$ ($I = 1, \dots, n$); for instance, $n = 3$ in $SO(10)$ and $n = 6$ in E_6 . [8] They will generally get superheavy Majorana masses denoted by an $n \times n$ mass matrix $(M_R)_{IJ}$, and also possesses the Dirac masses (R-L transition mass terms)

$$(M_D^T)_{iI} \sim y_\nu v \lambda^{h_u} \times (\lambda^{\psi_i(\mathbf{5}^*) + \psi_I^R}) \quad (25)$$

induced from

$$y_\nu \Psi_i(\mathbf{5}^*) \Psi_I(\mathbf{1}) H_u(\mathbf{5}) \left(\frac{\Theta}{M_{\text{Pl}}} \right)^{\psi_i(\mathbf{5}^*) + \psi_I^R + h_u}. \quad (26)$$

Here ψ_I^R denotes the $U(1)_X$ charges of the right-handed neutrinos $\Psi_I(\mathbf{1})$.

The Majorana mass matrix M_ν of (left-handed) neutrino is induced from these masses M_R and M_D by the see-saw mechanism [10] and evaluated as

$$\begin{aligned} (M_\nu)_{ij} &\sim (M_D^T)_{iI} (M_R^{-1})_{IJ} (M_D)_{Jj} \\ &\sim \lambda^{\psi_i(\mathbf{5}^*)} \left(\lambda^{\psi_I^R} (M_R^{-1})_{IJ} \lambda^{\psi_J^R} \right) \lambda^{\psi_j(\mathbf{5}^*)} \\ &\propto \lambda^{\psi_i(\mathbf{5}^*) + \psi_j(\mathbf{5}^*)}. \end{aligned} \quad (27)$$

Note that the dependence on the $U(1)_X$ charges ψ_I^R of the right-handed neutrinos has completely dropped out. (We should however take it into account that this occurs only for a generic case and may be broken in particular cases in which $(M_D^T)_{iI}$ brings about correlation between the left-handed neutrino index i and right-handed one I . [8]) Plugging the values (20) for $\psi_i(\mathbf{5}^*)$, we thus have

$$M_\nu \propto \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}. \quad (28)$$

This neutrino mass matrix happens to take the same form as one of the models that have been proposed by Ling and Ramond [11] and Babu, Gogoladze and Wang [12] This form is very interesting.

First, this matrix implies the large 2-3 mixing in the diagonalization matrix U_ν . The 2-3 mixing is also large in the charged lepton mixing matrix U_l as we have seen above, and so is it generally in the MNS matrix $U_{\text{MNS}} = U_l U_\nu^\dagger$ unless a cancellation occurs between U_l and U_ν .

Second, it is natural to assume that three neutrino masses are not accidentally degenerate. Then, the mass squared difference ratio (2) of the solar and atmospheric neutrinos implies the mass ratio of the second and third neutrinos: $m_{\nu 2}/m_{\nu 3} \sim \lambda$. In order for the M_ν to reproduce this mass ratio, the determinant of the 2×2 bottom-right submatrix of this M_ν should not be naturally-expected order 1, but should be $O(\lambda)$; that is, that submatrix should be diagonalized by an 2×2 unitary matrix u_ν as

$$u_\nu^* \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} u_\nu^\dagger \sim \begin{pmatrix} \lambda & 0 \\ 0 & 1 \end{pmatrix}. \quad (29)$$

If this is the case, the mass matrix M_ν takes the following form after the diagonalization of this 2×2 bottom-right submatrix:

$$\begin{pmatrix} 1 & 0 \\ 0 & u_\nu^* \end{pmatrix} M_\nu \begin{pmatrix} 1 & 0 \\ 0 & u_\nu^\dagger \end{pmatrix} \sim \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & \lambda & 0 \\ \lambda & 0 & 1 \end{pmatrix}. \quad (30)$$

If we note the 2×2 top-left submatrix of this matrix

$$\begin{pmatrix} \lambda^2 & \lambda \\ \lambda & \lambda \end{pmatrix}, \quad (31)$$

we see that this also gives the large mixing in the 1-2 sector so that it explains the *bi-large mixing*.

Therefore, the experimental fact

$$\frac{\Delta m_{\odot}^2}{\Delta m_{\oplus}^2} \sim \lambda^2 \quad \Leftrightarrow \quad \frac{m_{\nu 2}}{m_{\nu 3}} \sim \lambda \quad (32)$$

necessarily implies the bi-large mixing!

We note that a very similar neutrino mass matrix M_ν to ours (28) was also proposed by Maekawa: [13]

$$M_\nu \propto \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda^1 \\ \lambda^{1.5} & \lambda^1 & \lambda^{0.5} \\ \lambda^1 & \lambda^{0.5} & 1 \end{pmatrix}. \quad (33)$$

We should note that there is one more prediction in our framework; that is, it predicts a rather ‘large’ value for the element $U_{e3} \equiv (U_{\text{MNS}})_{13}$:

$$U_{e3} \sim O(\lambda^1) \quad \sim \quad (0.1 - 0.5). \quad (34)$$

This is seen as follows. First, for U_l , we have

$$\begin{aligned} (U_l)_{11} &\sim O(1), \\ (U_l)_{12} \text{ and } (U_l)_{13} &\sim \lambda^{\psi_1(\mathbf{5}^*) - \psi_2(\mathbf{5}^*)} = \lambda^1, \end{aligned} \quad (35)$$

which have resulted from down-type quark masses and an $SU(5)$ relation. Second, for U_ν , we have

$$\begin{aligned} (U_\nu)_{31} &\sim \lambda^{\psi_1(\mathbf{5}^*) - \psi_3(\mathbf{5}^*)} = \lambda^1, \\ (U_\nu)_{32} \text{ and } (U_\nu)_{33} &\sim O(1). \end{aligned} \quad (36)$$

These clearly give rise to $U_{e3} \equiv (U_{\text{MNS}})_{13} = (U_l U_\nu^\dagger)_{13} \sim O(\lambda)$. Although the bigger side of this prediction is already excluded experimentally, this prediction gives a crucial test for the idea of Froggatt-Nielsen mechanism.

Summarizing the points up to here, we have shown:

1. If we assume Froggatt-Nielsen’s factorized form for the quark/lepton mass matrices and the $SU(5)$ structure for the $U(1)_X$ charges, an input of up- and down-type quark masses necessarily implies that the 2-3 mixing is large in the MNS matrix U_{MNS} .

2. If we further add the data $\sqrt{\Delta m_{\odot}^2 / \Delta m_{\oplus}^2} \sim \lambda$, then, it implies that the 1-2 mixing in U_{MNS} is also large, so leading to bi-large mixing.

3. The measurement of U_{e3} will *confirm* or *kill* the basic idea of *Froggatt-Nielsen mechanism* for explaining the hierarchical mass structures of quarks and leptons.

Let us now look back our analysis and examine the possible implication of the neutrino data on the GUTs.

Recall first that the mixing unitary matrices are determined solely by the $U(1)$ charges of the left-handed components, and that the left-handed components fall into $SU(2)$ doublets under the standard gauge symmetry. Therefore, under the assumption of the generic factorized power form for the mass matrices, the standard gauge symmetry alone predicts

$$U_u \sim U_d \sim (\lambda^{|Q_i - Q_j|}), \quad U_\nu \sim U_l \sim (\lambda^{|L_i - L_j|}), \quad (37)$$

and hence also, for CKM and MNS matrices,

$$\begin{aligned} U_{\text{CKM}} &= U_u U_d^\dagger \sim (\lambda^{|Q_i - Q_j|}), \\ U_{\text{MNS}} &= U_l U_\nu^\dagger \sim (\lambda^{|L_i - L_j|}). \end{aligned} \quad (38)$$

Here Q_i and L_i are the $U(1)$ charges of the quark and lepton doublets, respectively. If we had the Pati-Salam symmetry $SU(4)_{\text{PS}}$, instead of the $SU(5)$ symmetry as assumed in the above, the $U(1)$ charges of quark and lepton doublets must be the same, $Q_i = L_i$, which leads to an incorrect prediction that the CKM and MNS mixing matrices should have the same structure, $U_{\text{CKM}} \sim U_{\text{MNS}}$.

This apparently seems to exclude GUT gauge groups like $SO(10)$ and E_6 larger than $SU(5)$, since they necessarily contain the $SU(4)_{\text{PS}}$ and the gauge multiplet members all carry a common $U(1)_X$ charge. Actually, any $SO(10)$ models in which three generations of matters come from three **16** representations are excluded, since there the leptons are the fourth colored ‘quarks’ of $SU(4)_{\text{PS}}$ and so the relations $Q_i = L_i$ must hold. The group E_6 , however, has two intrinsic mechanisms giving the ways out of this difficulty, and hence the E_6 models in which three generations come from three **27** representations are indeed allowed.

The first reason is because **27** is decomposed into the $SU(5)$ multiplets

$$\mathbf{27} = \underbrace{(\mathbf{10} + \mathbf{5}^* + \mathbf{1})}_{SO(10) \mathbf{16}} + \underbrace{(\mathbf{5} + \mathbf{5}^*)}_{SO(10) \mathbf{10}} + \underbrace{\mathbf{1}}_{SO(10) \mathbf{1}} \quad (39)$$

The point here is that there appear two $SU(5)$ $\mathbf{5}^*$ representations in each **27**, and six in all for three generation case. The $SU(4)_{\text{PS}}$ partner of the quark doublet contained in **10** in $SO(10)$ **16** is the lepton doublet in the $\mathbf{5}^*$ in $SO(10)$ **16**, but not that in the $\mathbf{5}^*$ in $SO(10)$ **10**. However, the light lepton doublets may generally come from any three (linear combinations) out of these six $SU(5)$ $\mathbf{5}^*$, and, so the relation $Q_i = L_i$ can easily be avoided. [8,14]

Second, it is also important that three **27** contain six $SU(5)$ singlets **1** which play the roll of right-handed neutrinos and form 6×3 Dirac mass matrix with the three left-handed neutrinos in the three light lepton doublets. However, depending on the $SO(10)$ properties of those three lepton doublets, there may appear zeros in the matrix elements of the 6×3 Dirac mass matrix. This is because, for instance, $SU(5)$ singlet in $SO(10)$ **16**, denoted $(\mathbf{1}, \mathbf{16})$ for brevity, can form Dirac masses with the left-handed neutrinos in $(\mathbf{5}^*, \mathbf{16})$ but not those in $(\mathbf{5}^*, \mathbf{10})$, when the Higgs is assumed to be $(\mathbf{5}, \mathbf{10})$. And 6×6 Majorana mass matrix for the right-handed neutrinos can have additional structure other than the Froggatt-Nielsen’s $U(1)$ power structure, since $(\mathbf{1}, \mathbf{16})$ and $(\mathbf{1}, \mathbf{1})$

have different $SO(10)$ quantum numbers and their Majorana masses may come from different Higgs scalars. These two facts invalidate the above discussion in Eq. (27) proving the factorization of the light neutrino mass matrix M_ν . So $U_\nu \sim (\lambda^{|L_i - L_j|})$ no longer remains true and $U_{\text{CKM}} \not\sim U_{\text{MNS}}$ even when $Q_i = L_i$.

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